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Scalar Higgs boson production in fusion of two off-shell gluons

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Abstract. The amplitude for scalar Higgs boson production in the fusion of two off-shell gluons is calculated including finite (not infinite) masses of quarks in the triangle loop. In comparison to the effective Lagrangian approach, we have found a new term in the amplitude. The matrix element found can be used in the $k_\perp$-factorization approach to the Higgs boson production. The results are compared with the calculations for on-shell gluons. Small deviations from the $\cos^2\phi$-dependence are predicted. The off-shell effects found are practically negligible.

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1 Introduction

The Higgs boson is the only missing, undiscovered component of the standard model of particle physics. Within the context of the standard model, the Higgs boson is responsible for the breaking of the $SU_\mathrm{L}(2)\times U(1)$ gauge symmetry and provides the mechanism for the generation of masses of the corresponding gauge bosons: $W^\pm$ and Z. In addition, the same mechanism provides masses for the leptons and quarks via Yukawa couplings. Therefore, the discovery and subsequent study of the Higgs boson properties is of the highest priority for particles physics community.

A precise theoretical understanding of Higgs production rate is critical to any attempts to search for the particle. The dominant production mechanism for Higgs bosons in hadron–hadron colliders is via gluon–gluon fusion [1] $pp\to gg\to H$, in which gluons fuse through a virtual top quark triangle to produce the Higgs boson. Such a process provides the largest production rate for the entire Higgs mass range of interest.

The standard description of hard processes in hadron collisions is within the framework of the QCD parton model, which reduces the hadron–hadron interactions to the parton–parton ones via the formalism of the hadron structure functions. The most popular approach is the QCD collinear approximation [2], based on the well known collinear factorization theorem [3]. In this approach all particles involved are assumed to be on the mass shell, carrying only longitudinal momenta, and the cross section is averaged over two transverse polarizations of the incident gluons. The transverse momenta of the incident partons are neglected in the QCD matrix elements. However, at small x, the effects of finite transverse momenta of partons become increasingly important, especially in the analysis of jets and heavy-quark production, and there is no reason to neglect the transverse momenta of the gluons in comparison to the quark mass. The method to incorporate the incident parton transverse momenta is referred to as the $k_\perp$-factorization approach [4, 5]. Here the Feynman diagrams are calculated taking into account the virtualities and all possible polarizations of the incident partons. There are widely discussed applications of the $k_\perp$-factorization approach to hard QCD processes like the J/ψ hadroproduction [6], charmonium production [7], heavy-quark photo- [8, 9] and hadroproduction [10], Higgs boson production [11, 12]. Some exclusive processes in the framework of the $k_\perp$-factorization approach are described in detail in [13].

In the lowest order the gluon coupling to the Higgs boson in the standard model is mediated by triangular loops with top quarks as shown by the Feynman diagram in Fig. 1. This process for on-shell gluons $p^2, k^2 = 0$ is well known in the literature [14], so we will only present the result:

$$T_{\mu\nu}|_{k^2,p^2=0} = \frac{\mathrm{i}\delta^{ab}\alpha_\mathrm{s}}{2\pi v}\left([(kp)g_{\mu\nu} - k_\mu p_\nu]I_1 + p_\mu k_\nu I_2\right), \tag{1}$$

$$I_1 = \tau\left[1 - \frac{1}{4}(1-\tau)\ln\left(\frac{\sqrt{1-\tau}+1}{\sqrt{1-\tau}-1}\right)^2\right], \tag{2}$$

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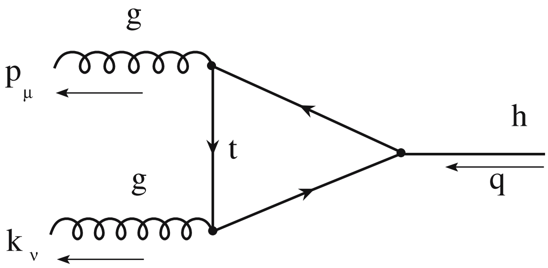

Fig. 1. Gluon coupling to the Higgs boson via a t-quark triangle loop

$$\text{for} \quad \tau = \frac{4m_{\rm top}^2}{m_h^2} > 1\,.$$

For a large quark mass $\tau \gg 1$ the form factor I_1 can be expanded in powers of τ^{-1}:

$$I_1 = \frac{2}{3}\left[1 + \frac{7}{30}\frac{1}{\tau} + \frac{2}{21}\frac{1}{\tau^2} + O\left(\frac{1}{\tau^3}\right)\right]\,,\; I_1|_{\tau\to\infty} = \frac{2}{3}\,. \tag{3}$$

The form factor I_2 is not relevant in the collinear approximation because it does not enter the squared matrix element in the on-shell limit.

In order to use the $k_\perp$-factorization approach we will take into account the non-trivial virtualities of the external gluons. The main technical problem here is to calculate the vector and second-rank tensor Feynman integrals with $p^2, k^2 \neq 0$. But the symmetric properties of the full amplitude together with a usage of convenient projectors allow one to avoid such difficulties and represent all relevant form factors in terms of scalar three- and two-point functions plus extra finite subtractions appearing due to the regularization.

In the present work we shall study in detail the fusion of off-shell gluons in proton–proton collisions producing a scalar Higgs boson by the top quark triangular loops with finite (not infinite) top quark mass. We analytically calculate the exact amplitude of this process in terms of two relevant form factors. The amplitude is studied in various kinematical regions. Furthermore we discuss different fermion mass limits. We find that the squared matrix element taking into account the non-zeroth gluon virtualities slightly (by several percents) increases with the growth of gluon transverse momentum. We also discuss the consequences of the non-zeroth virtualities for scalar boson production in the $k_\perp$-factorization approach for large transverse momenta.

2 The amplitude for the Higgs boson production in fusion of off-shell gluons

Let us start from the general tensor representation of the triangle amplitude shown in Fig. 1:

$$T_{\mu\nu}(k,p) = g_{\mu\nu}F_1 + k_\mu k_\nu F_2 + p_\mu p_\nu F_3 + k_\mu p_\nu F_4 + p_\mu k_\nu F_5\,. \tag{4}$$

Here $F_j = F_j(q^2, k^2, p^2, m_f^2)$, $j = 1, ..., 5$ are the Lorentz invariant form factors, m_f is the fermion mass in the loop. The Bose symmetry of the amplitude $T_{\mu\nu}(k,p) = T_{\nu\mu}(p,k)$ is equivalent to

$$F_1(k,p) = F_1(p,k)\,,\quad F_2(k,p) = F_3(p,k)\,,$$
$$F_4(k,p) = F_4(p,k)\,,\quad F_5(k,p) = F_5(p,k)\,. \tag{5}$$

The gauge invariance leads to the vector Ward identities $p^\mu T_{\mu\nu} = 0$, $k^\nu T_{\mu\nu} = 0$ which in terms of form factors gives

$$F_1 + p^2F_3 + (kp)F_4 = 0\,,\quad (kp)F_2 + p^2F_5 = 0\,,$$
$$F_1 + k^2F_2 + (kp)F_4 = 0\,,\quad (kp)F_3 + k^2F_5 = 0\,. \tag{6}$$

Therefore we have finally only the three relations

$$F_1 + p^2F_3 + (kp)F_4 = 0\,,$$
$$(kp)F_2 + p^2F_5 = 0\,, \tag{7}$$
$$p^2F_3 - k^2F_2 = 0\,.$$

This reduces the number of independent form factors to two. So the tensor representation of the amplitude can be expressed in the following form:

$$T_{\mu\nu}(k,p) = \mathrm{i}\delta^{ab}\frac{\alpha_{\rm s}}{2\pi}\frac{1}{v}\Bigg([(kp)g_{\mu\nu} - k_\mu p_\nu]G_1 \tag{8}$$
$$+\left[p_\mu k_\nu - \frac{p^2}{(kp)}k_\mu k_\nu - \frac{k^2}{(kp)}p_\mu p_\nu + \frac{k^2p^2}{(kp)^2}k_\mu p_\nu\right]G_2\Bigg),$$

where $\alpha_{\rm s}$ is the strong coupling constant, $v = (G_{\rm F}\sqrt{2})^{-1/2}$ is the electro-weak mass scale, and a, b are the color indices for the two off-shell gluons. This general form of the amplitude coincides with the standard expression for on-shell gluons (1) with

$$I_1 = G_1|_{k^2,p^2\to 0}\,,\quad I_2 = G_2|_{k^2,p^2\to 0}\,.$$

In order to calculate the form factors G_1, G_2 we introduce two projectors. The most convenient and simple choice is one symmetric $P_{\mu\nu} = g_{\mu\nu}$ and one antisymmetric $Q_{\mu\nu} = p_\mu k_\nu - p_\nu k_\mu$, both 2-rank tensors. The corresponding projections are

$$S_1 = T_{\mu\nu}Q^{\mu\nu} = \mathrm{i}\delta^{ab}\frac{\alpha_{\rm s}}{2\pi}\frac{1}{v}\Bigg([k^2p^2 - (kp)^2]G_1 + \left[2k^2p^2 - (kp)^2 - \frac{p^4k^4}{(kp)^2}\right]G_2\Bigg)\,, \tag{9}$$
$$S_2 = T_{\mu\nu}P^{\mu\nu} = \mathrm{i}\delta^{ab}\frac{\alpha_{\rm s}}{2\pi}\frac{1}{v}\left(3(kp)G_1 + \left[(kp) - \frac{k^2p^2}{(kp)}\right]G_2\right)\,.$$

Solving this system of equations with respect to G_1, G_2 we get

$$G_1 = [\mathrm{i}\delta^{ab}]^{-1}\frac{\pi v}{\alpha_{\rm s}}\frac{(kp)S_1 + ((kp)^2 - k^2p^2)S_2}{(kp)((kp)^2 - k^2p^2)}\,,$$
$$G_2 = -[\mathrm{i}\delta^{ab}]^{-1}\frac{\pi v}{\alpha_{\rm s}}\frac{(kp)(3(kp)S_1 + ((kp)^2 - k^2p^2)S_2)}{((kp)^2 - k^2p^2)^2}\,. \tag{10}$$

The explicit expression for the amplitude reads

$$T_{\mu\nu}(k,p) = \mathrm{i}\delta^{ab}\frac{4\pi\alpha_s}{v}\, m_f \times \int \frac{\mathrm{d}^4 r}{\mathrm{i}(2\pi)^4}\frac{M_{\mu\nu}}{\left[(r-k)^2-m_f^2\right]\left[r^2-m_f^2\right]\left[(r+p)^2-m_f^2\right]}\,, \quad (11)$$

$$M_{\mu\nu} = \mathrm{Tr}[(m_f + \not{r} - \not{k})\gamma_\nu(m + \not{r})\gamma_\mu(m_f + \not{r} + \not{p})]\,.$$

Employing now the dimensional regularization and using the Passarino–Veltman reduction we get

$$S_1 = \mathrm{i}\delta^{ab}\frac{\alpha_s}{\pi v}\frac{m_f^2}{\mu^{4-n}}\Big[C_0(kp)(k^2+p^2+2(kp)) - (k^2+(kp))B_0^{12} + (k^2+p^2+2(kp))B_0^{13} - - (p^2+(kp))B_0^{23}\Big]\,, \quad (12)$$

$$S_2 = \mathrm{i}\delta^{ab}\frac{\alpha_s}{\pi v}\frac{m_f^2}{\mu^{4-n}}\Big[C_0(4m^2-k^2-p^2-n(kp)) + B_0^{12} + (2-n)B_0^{13} + B_0^{23}\Big]\,, \quad (13)$$

where $n = 4-\varepsilon$ is the dimension of the space, B^{12}, B^{13}, B^{23} are the scalar two-point functions

$$B_0^{12}(k^2,m_f^2) = 16\pi^2\mu^{4-n} \times \int \frac{\mathrm{d}^n r}{\mathrm{i}(2\pi)^4}\frac{1}{\left[(r-k)^2-m_f^2\right]\left[r^2-m_f^2\right]} = \frac{2}{\varepsilon} + \xi - L_1\,,$$

$$B_0^{13}\left(q^2,m_f^2\right) = 16\pi^2\mu^{4-n} \times \int \frac{\mathrm{d}^n r}{\mathrm{i}(2\pi)^4}\frac{1}{\left[(r-k)^2-m_f^2\right]\left[(r+p)^2-m_f^2\right]} = \frac{2}{\varepsilon} + \xi - L_2\,,$$

$$B_0^{23}\left(p^2,m_f^2\right) = 16\pi^2\mu^{4-n} \times \int \frac{\mathrm{d}^n r}{\mathrm{i}(2\pi)^4}\frac{1}{\left[r^2-m_f^2\right]\left[(r+p)^2-m_f^2\right]} = \frac{2}{\varepsilon} + \xi - L_3\,,$$

where

$$L_j = \beta_j \ln\frac{\beta_j+1}{\beta_j-1}\,, \quad \beta_1 = \sqrt{1-\frac{4m_f^2}{k^2}}\,,$$

$$\beta_2 = \sqrt{1-\frac{4m_f^2}{q^2}}\,, \quad \beta_3 = \sqrt{1-\frac{4m_f^2}{p^2}}\,,$$

$$\xi = -\gamma - \ln\pi + 2 - \ln\frac{m_f^2}{\mu^2}\,,$$

and C_0 is the convergent scalar three-point function. Following [15] we have

$$C_0\left(q^2,k^2,p^2,m_f^2\right) = 16\pi^2\mu^{4-n} \times \int \frac{\mathrm{d}^n r}{\mathrm{i}(2\pi)^4}\frac{1}{\left[(r-k)^2-m_f^2\right]\left[r^2-m_f^2\right]\left[(r+p)^2-m_f^2\right]} = \kappa(k^2,p^2,q^2) + \kappa(q^2,k^2,p^2) + \kappa(p^2,q^2,k^2)\,.$$

Above for simplicity we have introduced the following notation:

$$\kappa(x,y,z) = \frac{1}{\lambda}\left[\mathrm{Li}_2\left(\frac{t-1}{t-\tau}\right) + \mathrm{Li}_2\left(\frac{t-1}{t+\tau}\right) - \mathrm{Li}_2\left(\frac{t+1}{t-\tau}\right) - \mathrm{Li}_2\left(\frac{t+1}{t+\tau}\right)\right]\,,$$

$$\lambda(x,y,z) = \sqrt{x^2+y^2+z^2-2xy-2yz-2zx}\,,$$

$$t = \frac{1}{\lambda}(x-y-z)\,, \quad \tau = \sqrt{1-\frac{4m_f^2}{x}}\,.$$

Of course, the projections S_1 and S_2 are convergent in the limit $\varepsilon \to 0$ and then

$$S_1 = \mathrm{i}\delta^{ab}\frac{\alpha_s}{\pi v}m_f^2\left[C_0(kp)(k^2+p^2+2(kp)) + ((kp)+k^2)L_1 - (k^2+p^2+2(kp))L_2 + ((kp)+p^2)L_3\right]\,, \quad (14)$$

$$S_2 = \mathrm{i}\delta^{ab}\frac{\alpha_s}{\pi v}m_f^2\left[C_0\left(4m_f^2-k^2-p^2-4(kp)\right) - L_1 + 2L_2 - L_3 + 2\right]\,. \quad (15)$$

There is a finite subtraction term in S_2 as a consequence of regularization. Substituting these expressions into (10) we finally obtain

$$G_1 = \frac{m_f^2}{(kp)((kp)^2-k^2p^2)}\big[\big(4m_f^2((kp)^2-k^2p^2) - 2(kp)((kp)^2-2k^2p^2) + k^2p^2(k^2+p^2)\big)ptC_0 + k^2(p^2+(kp))L_1 - (2k^2p^2+(kp)(k^2+p^2))L_2 + p^2(k^2+(kp))L_3 + 2((kp)^2-k^2p^2)\big]\,, \quad (16)$$

$$G_2 = -\frac{m_f^2(kp)}{((kp)^2-k^2p^2)^2}\big[\big(4m_f^2((kp)^2-k^2p^2) + (k^2+p^2)(2(kp)^2+k^2p^2) + 2(kp)((kp)^2+2k^2p^2)\big)C_0 + (2(kp)^2+3(kp)k^2+k^2p^2)L_1 - (3(kp)(k^2+p^2) + 2(2(kp)^2+k^2p^2))L_2 + (2(kp)^2+3(kp)p^2+k^2p^2)L_3 + 2((kp)^2-k^2p^2)\big]\,. \quad (17)$$

Now the off-shell amplitude can be calculated as $M = T_{\mu\nu}k_{1\perp}^{\mu}k_{2\perp}^{\nu}/|\mathbf{k}_{1\perp}||\mathbf{k}_{2\perp}|$ with $T_{\mu\nu}$ given by (8) and G_1 and G_2 given by (16) and (17), respectively.

3 Estimating the size of the off-shell effects

In the so-called heavy fermion limit $m_f \to \infty$ we get

$$G_1|_{m_f\to\infty} = \frac{2}{3}, \quad G_2|_{m_f\to\infty} = 0 \tag{18}$$

for gluons with arbitrary virtualities k^2 and p^2. The first limit for G_1 coincides with the classical on-shell result (3).

It is easy to show also that the form factor G_1 in the limit $k^2, p^2 \to 0$ is

$$G_1|_{k^2,p^2\to 0} = \tau\left[1-\frac{1}{4}(1-\tau)\ln\left(\frac{\sqrt{1-\tau}+1}{\sqrt{1-\tau}-1}\right)^2\right]$$
$$\text{for} \quad \tau = \frac{4m_f^2}{q^2} > 1\,, \tag{19}$$

which coincides with I_1, see (2), for the relevant case of Higgs boson production from on-shell gluon fusion with $q^2 = (k+p)^2 \simeq m_h^2$, $m_f = m_{\rm top}$. In this limit the second form factor G_2 is

$$G_2|_{k^2,p^2\to 0} = -\tau\left[5-2\sqrt{1-\tau}\ln\left(\frac{\sqrt{1-\tau}+1}{\sqrt{1-\tau}-1}\right)\right.$$
$$\left.+\frac{1}{4}(1+\tau)\ln\left(\frac{\sqrt{1-\tau}+1}{\sqrt{1-\tau}-1}\right)^2\right]. \tag{20}$$

Expansions for $\tau \gg 1$ and p^2, $k^2 = 0$ have the following form:

$$G_1 = \frac{2}{3}\left[1+\frac{7}{30}\frac{1}{\tau}+\frac{2}{21}\frac{1}{\tau^2}+O\left(\frac{1}{\tau^3}\right)\right],$$
$$G_2 = -\frac{1}{45}\frac{1}{\tau}-\frac{4}{315}\frac{1}{\tau^2}+O\left(\frac{1}{\tau^3}\right). \tag{21}$$

Let us write analogous expansions taking into account the non-zeroth gluon virtualities.

The infinitely heavy fermion limits (18) do not contain at all the dimensional quantities such as q^2, p^2, k^2, so there is no difference in the order of limits: at first $q^2 \to 0$ and then k^2, $p^2 \to 0$ or vice versa. Therefore it is more convenient to work with the dimensionless parameters defined as

$$\chi = \frac{q^2}{4m_f^2}\,, \qquad \xi = \frac{p^2}{4m_f^2} < 0\,, \qquad \eta = \frac{k^2}{4m_f^2} < 0\,. \tag{22}$$

Then the on-shell limit $p^2, k^2 \to 0$ is equivalent to ξ, $\eta \to 0$. The heavy-quark limit (18) corresponds to χ, ξ, $\eta \to 0$. We can now take into account the terms of first order in ξ and η in (21). In the case of Higgs boson production on average $m_h^2 \gg |\langle k^2\rangle|, |\langle p^2\rangle|$ [12] so we have to take into account the powers of χ higher than the powers of ξ, η. Expansions of G_1 and G_2 in χ up to second order and ξ and η to first order give

$$G_1(\chi,\xi,\eta) = \frac{2}{3}\left[1+\frac{7}{30}\chi+\frac{2}{21}\chi^2\right.$$
$$\left.+\frac{11}{30}(\xi+\eta)+O\left(\chi^3,\,\xi^2,\,\eta^2,\,\chi\xi,\,\chi\eta,\,\xi\eta\right)\right], \tag{23}$$
$$G_2(\chi,\xi,\eta) = -\frac{1}{45}(\chi-\xi-\eta)$$
$$-\frac{4}{315}\chi^2+O\left(\chi^3,\,\xi^2,\,\eta^2,\,\chi\xi,\,\chi\eta,\,\xi\eta\right).$$

We have compared the exact form factors (16) and (17) with their expansion counterparts (23) for realistic parameters $m_{\rm top} = 0.17\,$TeV, $m_h = 0.15\,$TeV. In conclusion, we can use the form factor expansions up to $|\xi|$, $|\eta| = 0.3$ with a maximal error of less than 1%.

To make our calculations useful for the case of Higgs boson production let us turn to the physical parameters relevant for proton–proton collisions. Introducing $p = x_1p_1 + k_{1\perp}$, $k = x_2p_2 + k_{2\perp}$, where $k_{1\perp}$, $k_{2\perp}$ are space-like four-vectors associated with the transverse momenta of gluons $\mathbf{k}_{1\perp}$ and $\mathbf{k}_{1\perp}$, p_1, p_2 are the hadron momenta, so $(p_1p_2) = s/2$, and neglecting the hadron mass $m_p \ll m_h$, $m_{\rm top}$, we see that $p^2 \simeq -\mathbf{k}_{1\perp}^2 < 0$ and $k^2 \simeq -\mathbf{k}_{2\perp}^2 < 0$. The subject of our analysis is the normalized projection of the amplitude $M = T_{\mu\nu}k_{1\perp}^\mu k_{2\perp}^\nu/|\mathbf{k}_{1\perp}||\mathbf{k}_{2\perp}|$, given by the formula

$$M(g^*g^*\to H) = -\mathrm{i}\delta^{ab}\frac{\alpha_{\rm s}}{4\pi}\frac{1}{v}$$
$$\times\left[\left(m_h^2+\mathbf{k}_{1\perp}^2+\mathbf{k}_{2\perp}^2+2|\mathbf{k}_{1\perp}||\mathbf{k}_{2\perp}|\cos\phi\right)\cos\phi\, G_1\right. \tag{24}$$
$$\left.-\frac{2\left(m_h^2+\mathbf{k}_{1\perp}^2+\mathbf{k}_{2\perp}^2+2|\mathbf{k}_{1\perp}||\mathbf{k}_{2\perp}|\cos\phi\right)^2|\mathbf{k}_{1\perp}||\mathbf{k}_{2\perp}|}{\left(m_h^2+\mathbf{k}_{1\perp}^2+\mathbf{k}_{2\perp}^2\right)^2}G_2\right],$$

where G_1, G_2 can be taken from (23) for not too large $|\mathbf{k}_{1\perp}|$ and $|\mathbf{k}_{2\perp}|$, ϕ is the azimuthal angle between the gluon transverse momenta $\mathbf{k}_{1\perp}$ and $\mathbf{k}_{2\perp}$, the transverse momentum of the produced Higgs boson is $\mathbf{p}_\perp = \mathbf{k}_{1\perp}+\mathbf{k}_{2\perp}$, and the virtual gluon polarization tensor has been taken in the form [5, 11]

$$\sum \epsilon^\mu\epsilon^{*\,\nu} = \frac{k_\perp^\mu k_\perp^\nu}{\mathbf{k}_\perp^2}\,.$$

For brevity, we shall denote the first term of the amplitude by $\mathcal{M}_1$ and the second term by $\mathcal{M}_2$. Neglecting the second term in (24) and taking the on-shell value of G_1, one obtains

$$M_1 = -\mathrm{i}\delta^{ab}\frac{\alpha_{\rm s}}{4\pi}\frac{1}{v}\left(m_h^2+\mathbf{k}_{1\perp}^2+\mathbf{k}_{2\perp}^2+2|\mathbf{k}_{1\perp}||\mathbf{k}_{2\perp}|\cos\phi\right)$$
$$\times\cos\phi\, I_1\,,$$
$$I_1 = G_1|_{k^2,p^2\to 0} \simeq G_1^0\,. \tag{25}$$

This coincides with the amplitude obtained in [11] within a numerical factor.

Let us now quantify some of the off-shell effects discussed above.

3.1 Effect on form factors

Let us start with a simple case of form factors. In Figs. 2 and 3 we show the dependence of the two off-shell

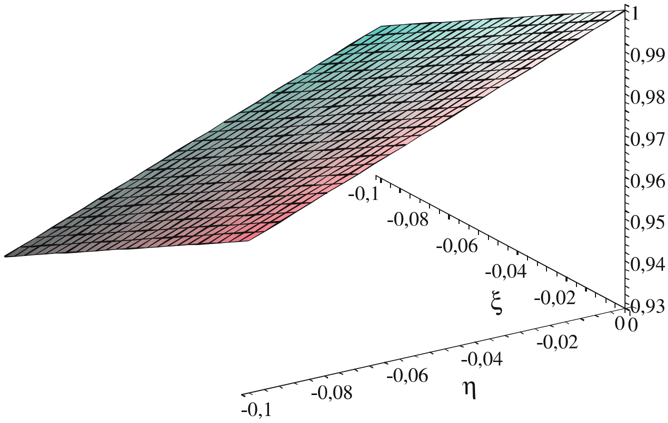

Fig. 2. Off-shell form factor G_1 normalized to its on-shell value, as a function of $\xi = -\mathbf{k}_{1\perp}^2/4m_{\rm top}^2$ and $\eta = -\mathbf{k}_{2\perp}^2/4m_{\rm top}^2$

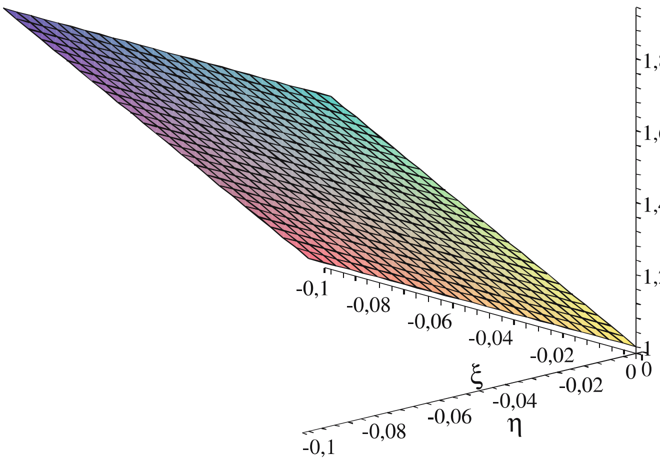

Fig. 3. Off-shell form factor G_2 normalized to its on-shell value, as a function of $\xi = -\mathbf{k}_{1\perp}^2/4m_{\rm top}^2$ and $\eta = -\mathbf{k}_{2\perp}^2/4m_{\rm top}^2$

form factors G_1 and G_2 on the parameters ξ and η. The results are normalized to the on-shell values of the form factors,

$$G_1^0 = \frac{2}{3}\left[1 + \frac{7}{30}\chi + \frac{2}{21}\chi^2\right], \qquad (26)$$
$$G_2^0 = -\frac{1}{45}\chi - \frac{4}{315}\chi^2 .$$

In this calculation $m_{\rm top} = 0.17$ TeV, $m_h = 0.15$ TeV. We see that the first form factor G_1 slightly drops, while the second form factor G_2 grows with increasing $|\xi|$ and $|\eta|$. The first form factor with taking into account of the finite quark mass differs from the one obtained in the framework of the effective approach [11] by 5%.

3.2 Effect on amplitude

By averaging the amplitude squared M^2 over ϕ we obtain

$$\langle M^2\rangle_\phi = \frac{\alpha_s^2}{4\pi^2}\frac{1}{v^2}\Bigg[\left(\left(m_h^2 + \mathbf{k}_{1\perp}^2 + \mathbf{k}_{2\perp}^2\right)^2 + 2\,\mathbf{k}_{1\perp}^2\mathbf{k}_{2\perp}^2\right) G_1^2$$
$$+ \frac{\left[\left(\left(m_h^2 + \mathbf{k}_{1\perp}^2 + \mathbf{k}_{2\perp}^2\right)^2 + 6\,\mathbf{k}_{1\perp}^2\mathbf{k}_{2\perp}^2\right)^2 - 32\,\mathbf{k}_{1\perp}^4\mathbf{k}_{2\perp}^4\right]}{\left(m_h^2 + \mathbf{k}_{1\perp}^2 + \mathbf{k}_{2\perp}^2\right)^4}$$
$$\times 8\mathbf{k}_{1\perp}^2\mathbf{k}_{2\perp}^2 G_2^2 \qquad (27)$$
$$- \frac{\left[3\,\left(m_h^2 + \mathbf{k}_{1\perp}^2 + \mathbf{k}_{2\perp}^2\right)^2 + 2\,\mathbf{k}_{1\perp}^2\mathbf{k}_{2\perp}^2\right]}{\left(m_h^2 + \mathbf{k}_{1\perp}^2 + \mathbf{k}_{2\perp}^2\right)^2}$$
$$\times 8\mathbf{k}_{1\perp}^2\mathbf{k}_{2\perp}^2 G_1 G_2\Bigg] .$$

One can see that in the collinear limit $\mathbf{k}_{1\perp}, \mathbf{k}_{2\perp} \to 0$ the averaged square of the matrix element, $\langle M^2\rangle_\phi$, coincides with the squared matrix element in the covariant gauge, $T^{\mu\nu}T^{\mu'\nu'}\, g_{\mu\mu'}g_{\nu\nu'}$, multiplied by 2.

Let us define the matrix element M_0 that is obtained from M given by (24) by substituting the off-shell form factors by the on-shell ones G_1^0 and G_2^0 (see (27)):

$$M_0 = -{\rm i}\delta^{ab}\frac{\alpha_s}{4\pi}\frac{1}{v}\left[\left(m_h^2 + \mathbf{k}_{1\perp}^2 + \mathbf{k}_{2\perp}^2 + 2|\mathbf{k}_{1\perp}||\mathbf{k}_{2\perp}|\cos\phi\right)\right.$$
$$\times \cos\phi\, G_1^0 \qquad (28)$$
$$- \frac{\left(m_h^2 + \mathbf{k}_{1\perp}^2 + \mathbf{k}_{2\perp}^2 + 2|\mathbf{k}_{1\perp}||\mathbf{k}_{2\perp}|\cos\phi\right)^2}{\left(m_h^2 + \mathbf{k}_{1\perp}^2 + \mathbf{k}_{2\perp}^2\right)^2}$$
$$\left.\times 2\,|\mathbf{k}_{1\perp}||\mathbf{k}_{2\perp}|G_2^0\right] ,$$

In Fig. 4 we show $\langle M^2\rangle_\phi/\langle M_0^2\rangle_\phi$ as a function of the reduced parameters ξ and η. One can see a drop of $\langle M^2\rangle_\phi$ relative to $\langle M_0^2\rangle_\phi$ due to the drop of the form factor G_1.

Let us first estimate the off-shell effect. Following [12] we take the values of the transverse gluon momenta in the interval $|\mathbf{k}_{1,2\perp}| \sim 5$–$50$ GeV. Then the relative effect of replacing G_1 and G_2 by their on-shell counterparts G_1^0 and G_2^0 is

$$\frac{\langle M_0^2\rangle_\phi - \langle M^2\rangle_\phi}{\langle M_0^2\rangle_\phi} \simeq \frac{(G_1^0)^2 - G_1^2}{(G_1^0)^2} = 0.0003\text{–}0.03\,, \qquad (29)$$

respectively for the lower and upper limits of the gluon transverse momenta. A rather small drop of the averaged

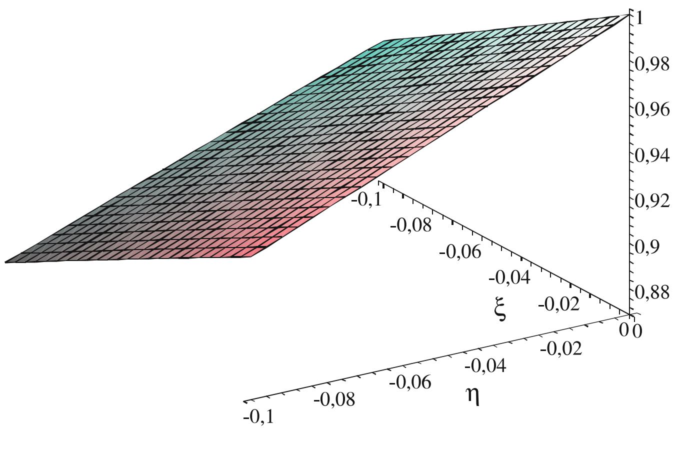

Fig. 4. Averaged square of off-shell matrix element $\langle M^2\rangle_\phi$ normalized to its on-shell value $\langle M_0^2\rangle_\phi$ as a function of $\xi = -\mathbf{k}_{1\perp}^2/4m_{\rm top}^2$ and $\eta = -\mathbf{k}_{2\perp}^2/4m_{\rm top}^2$

squared matrix element can be observed, mainly due to the drop of the first form factor G_1. The effect is of the order of 1% or less at typical gluon transverse momenta. Therefore the effect of non-zeroth gluon virtualities in the form factors on the averaged squared matrix element is rather small.

It seems interesting to study the behavior of the matrix element in this part of the phase space where the first term is small. Actually, when $\cos\phi = 0$ the contribution of the form factor G_1 disappears from (24), and we have the following simple expression for the amplitude:

$$M|_{\phi=\pi/2} = \mathrm{i}\delta^{ab}\frac{\alpha_s}{2\pi}\frac{1}{v}|\mathbf{k}_{1\perp}||\mathbf{k}_{2\perp}|\,G_2\,, \qquad (30)$$

which is determined exclusively by the second form factor G_2. As a result there are significant consequences of the non-zeroth virtualities on the angular distribution around $\phi = \pi/2$. Then $M^2|_{\phi=\pi/2}$ grows with increasing $|\xi|$ and $|\eta|$. The relative growth is determined completely by the growth of the form factor G_2:

$$\frac{M^2|_{\phi=\pi/2} - M_0^2|_{\phi=\pi/2}}{M_0^2|_{\phi=\pi/2}} \simeq \frac{{G_2}^2 - {G_2^0}^2}{{G_2^0}^2} = 0.004\text{–}0.44$$

at $|\mathbf{k}_{1,2\perp}| \sim 5$–$50\,\mathrm{GeV}$, respectively. Thus taking into account the non-zeroth virtualities could considerably change the angular distribution at $\phi \approx \pi/2$. Whether this can be observed in reality will be discussed in the next section.

3.3 Effect on inclusive cross section

Let us try to evaluate how big can be the observable effect. This requires a convolution of the off-shell subprocess cross section with realistic unintegrated gluon distributions. The inclusive cross section for the production of the Higgs boson in hadron–hadron collisions can be written as

$$\sigma = \int \frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\frac{\mathrm{d}^2\mathbf{k}_{1\perp}}{\pi}\frac{\mathrm{d}^2\mathbf{k}_{2\perp}}{\pi} \times \sigma_{\text{off-shell}}\left(\mathbf{k}_{1\perp},\mathbf{k}_{2\perp}\right)\,\mathcal{A}\left(x_1,\mathbf{k}_{1\perp}^2,\mu_F^2\right)\,\mathcal{A}(x_2,\mathbf{k}_{2\perp}^2,\mu_F^2)\,,$$

where the $\mathcal{A}(x,\mathbf{k}_\perp^2,\mu^2)$ are the unintegrated gluon distributions in the colliding hadrons h_1 and h_2.

Now the distribution in azimuthal angle ϕ between $\mathbf{k}_{1\perp}$ and $\mathbf{k}_{2\perp}$ can be calculated as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\phi} = 2\pi\int\frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\frac{1}{\pi^2}k_{1\perp}\mathrm{d}k_{1\perp}k_{2\perp}\mathrm{d}k_{2\perp} \times \sigma_{\text{off-shell}}\left(\mathbf{k}_{1\perp},\mathbf{k}_{2\perp}\right)\,\mathcal{A}\left(x_1,\mathbf{k}_{1\perp}^2,\mu^2\right)\,\mathcal{A}(x_2,\mathbf{k}_{2\perp}^2,\mu^2).$$

Inserting the off-shell matrix element squared into the formula for the off-shell subprocess cross section we get

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\phi} = \frac{\alpha_s^2\left(\mu^2\right)}{256\,\pi^2}\,\frac{\left(m_h^2+\mathbf{p}_\perp^2\right)}{v^2x_1x_2\,s\,m_h^2} \times\int\left[\cos\phi\,G_1 - \frac{2\left(m_h^2+\mathbf{p}_\perp^2\right)|\mathbf{k}_{1\perp}||\mathbf{k}_{2\perp}|}{\left(m_h^2+\mathbf{k}_{1\perp}^2+\mathbf{k}_{2\perp}^2\right)^2}\,G_2\right]^2 \times\mathcal{A}\left(x_1,\mathbf{k}_{1\perp}^2,\mu^2\right)\mathcal{A}\left(x_2,\mathbf{k}_{2\perp}^2,\mu^2\right)\,\mathrm{d}\,\mathbf{k}_{1\perp}^2\mathrm{d}\,\mathbf{k}_{2\perp}^2\mathrm{d}y_{\mathrm{H}}\,, \qquad (31)$$

where y_{H} is the center-of-mass Higgs boson rapidity and the longitudinal momentum fractions must be evaluated as

$$x_1 = \sqrt{\frac{m_h^2+\mathbf{p}_\perp^2}{s}}\exp(y_{\mathrm{H}})\,, \quad x_2 = \sqrt{\frac{m_h^2+\mathbf{p}_\perp^2}{s}}\exp(-y_{\mathrm{H}})\,.$$

The expression (31), in the on-shell limit for the form factors, is consistent with the analogous formula obtained by Lipatov and Zotov in [11].

The second term of the off-shell amplitude (24) is new and has not been discussed so far in the literature. For illustrating the role of the second part of the amplitude in Fig. 5 we show the azimuthal angle distributions. In this calculation the BFKL unintegrated gluon distributions were used for example (for more details see e.g. [12]). There is only a small difference between the result obtained with the sum of both amplitudes ($\mathcal{M}_1+\mathcal{M}_2$) and the result obtained with the first term ($\mathcal{M}_1$) only. The difference becomes visible only close to $\phi = \pi/2$, i.e. when the first amplitude vanishes. We show in the figure also the modulus of the interference term. The latter is much smaller than the cross section of the first term only, except for $\phi \approx \pi/2$. Because this effect is negligible except when extremely close to $\phi = \pi/2$, we see no easy way to identify the off-shell effects.

From the theoretical point of view the exact off-shell matrix element would break the familiar $\cos^2\phi$-dependence of the cross section. The second term of our exact matrix element leads to an asymmetry around $\pi/2$:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\phi}\left(\frac{\pi}{2}-\phi_0\right) < \frac{\mathrm{d}\sigma}{\mathrm{d}\phi}\left(\frac{\pi}{2}+\phi_0\right)\,, \qquad (32)$$

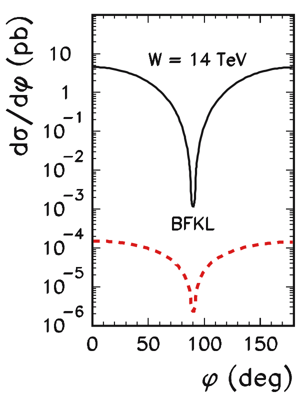

Fig. 5. Azimuthal angle distribution of the cross section. In this calculation the BFKL UGDF was used and $-2 < y_{\mathrm{H}} < 2$. The *solid line* represents the calculation with the full amplitude, whereas the *dashed line* is the modulus of the interference term

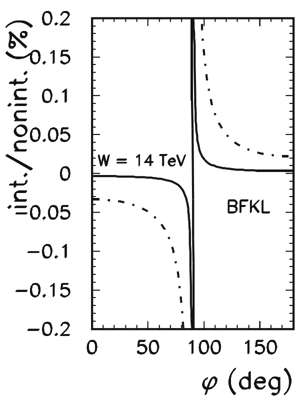

Fig. 6. The asymmetry term normalized to the symmetric terms as a function of ϕ. In this calculation the BFKL UGDF was used and $-2 < y_{\mathrm{H}} < 2$. The *solid curve* corresponds to the inclusive case, while the *dash-dotted curve* is for the extra cut $|\mathbf{p}_\perp| > 50\,\mathrm{GeV}$

for $\phi_0 > 0$. In Fig. 6 we show the ratio of the interference term to the sum of the two noninterference terms. Here the off-shell effect is only at the 10^{-3} level. A bigger effect is obtained at $\phi \approx \pi/2$. A change of sign at $\phi = \pi/2$ is clearly visible. Putting an extra cut on $|\mathbf{p}_\perp|$ could increase the relative asymmetry. In Fig. 6 we show the ratio obtained with such an extra cut $|\mathbf{p}_\perp| > 50$ GeV by the dash-dotted line.

4 Conclusion

In the present work we have analyzed the effect of the non-zeroth virtualities of external gluons on the amplitude of scalar Higgs boson production. The off-shell matrix element for this process was calculated. We have found a new term in the amplitude compared to a recent effective Lagrangian calculation.

A straightforward application of our analysis is in the case of inclusive Higgs boson production in proton–proton collisions at LHC. We have estimated that the relative drop of the averaged square of the matrix element caused by replacing the on-shell form factors by the off-shell ones is only about 1% or less at relevant physical parameters of the future experiments, so this effect could be verified in the high-precision experiments only. However, the effect of taking into account the non-zeroth virtualities on the angular distribution at $\phi \approx \pi/2$ is much more significant due to the quick growth of the second form factor G_2 as a function of gluon transverse momenta. The relative growth of the squared matrix element at $\phi = \pi/2$ is up to 45% for $|\mathbf{k}_{1,2\perp}| \sim 50$ GeV.

The observable effect was estimated in the framework of the $k_\perp$-factorization approach by convoluting the squared off-shell matrix element and the unintegrated gluon distributions. The effect found is, however, extremely small and concentrated around $\phi = \pi/2$. This will be extremely difficult to identify in the future rather low-statistics experiments.

We have discussed deviations from the $\cos^2\phi$-dependence of the cross section. A small asymmetry around $\phi = \pi/2$, at the 10^{-3} level, was found. Taking into account that the ϕ-dependence is not directly a measurable quantity the predicted effect seems extremely difficult to be identified experimentally.

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